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Stopping light in an active medium

Nan Sun[1], Jiangwei Chen[1,2,a], and Dongming Tang[3]

[1] College of Electronic Science and Engineering, Nanjing, University of Posts & Telecommunications, Nanjing 210046, P.R. China
[2] Key Laboratory of Radio Frequency and Micro-Nano Electronics of Jiangsu Province, Nanjing 210003, P.R. China
[3] College of Electronic Science and Engineering, Nanjing University, Nanjing 210093, P.R. China

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Abstract. On the basis of work done by Lorentz force and Maxwell's equations, energy conversion relationships of electromagnetic fields in the dispersive and absorptive media are analyzed. Then new definitions of stored and lossy energy densities are proposed, respectively. Furthermore, it is predicted that light may stop steadily in an active medium when both real part of impedance and imaginary part of wave vector equal zero, which corresponds to a case that time-dependent Poynting vector shift forward and then turn backward periodically.

1 Introduction

Slowing or even stopping light is one of the most interesting issues to the modern physical sciences and correlated technologies [1,2]. Recently, Heinze et al. have stopped light for a whole minute by using a technique called electromagnetically induced transparency [1]. Tsakmakidis et al. have proposed theoretically another way to stop light by applying an axially varying hetero-structure with a metamaterial core of negative refractive index [2]. These researches may lead to applications in optical data processing and storage or realization of optical memories.

Energy flow velocity is an essential parameter adopted to describe propagation properties of light. In the dispersive and absorptive media, knowledge of stored electromagnetic energy densities is important to address issues associated with energy flow velocity. Theoretical prediction [3] and experimental verification [4] of existence of left-handed material (LHM) invigorate researches in metamaterials and provide opportunities to reconsider the basic concepts and theorems of electromagnetic waves in lossy or gain media [5–11] (Gain may be taken as negative loss, below, we shall not distinguish difference between gain and loss unless otherwise indicated). Early, theoretical evaluation of electromagnetic energy density in a medium which is both electrically and magnetically dispersive and absorptive has been attempted by Askne and Lind [12]. However, their energy density expression is inappropriate for LHM, since it yields negative energy values near the resonance frequency of either permittivity or permeability [5]. Adopting equations of motion of electric and magnetic polarization, another type of definition of stored and lossy energy densities has been proposed [6,7], in which electric (magnetic) lossy energy density is directly taken as the term related to the damping one of the equation of motion of electric (magnetic) polarization without a rigorous classification criterion.

[a] e-mail: jwchen69@sohu.com

In this work, we shall offer a criterion to classify stored and lossy energies in the dispersive and absorptive media, and then investigate properties of energy flow velocity of electromagnetic wave in lossy media. It is predicted that, under certain conditions, electromagnetic wave may be stopped steadily in an active medium. To the best of our knowledge, this is a new possible approach to stop light. The remainder of the paper is organized as follows: in Section 2, a criterion to classify stored and lossy energies in lossy media is deduced from Maxwell's equations with Lorentz force considered. Thus new definitions of stored and lossy energy densities are proposed. In Section 3, the properties of energy flow velocity are discussed by applying the new definitions of stored and lossy energy densities Finally, some conclusions are drawn in Section 4.

2 Analysis on stored and lossy energies in a lossy medium

Let us begin by considering energy conversion relationships of electromagnetic fields in a dispersive and absorptive medium. For simplicity, we limit our attention to the homogeneous isotropic linear media. Choosing time dependence $e^{i\omega t}$, the medium can be generally represented by a complex scalar relative permittivity $\tilde{\varepsilon} = |\tilde{\varepsilon}| \exp(-i\alpha_\varepsilon) = \varepsilon' - i\varepsilon''$ and permeability $\tilde{\mu} = |\tilde{\mu}| \exp(-i\alpha_\mu) = \mu' - i\mu''$, respectively (In this paper, the complex valued parameters are marked with "∼"). $\alpha_{\varepsilon(\mu)}$ is electric (magnetic) damping angle. For passive media, both α_ε and α_μ are in the range of $[0, \pi]$, and for active media, either α_ε or α_μ may be in the range of $(\pi, 2\pi)$ [10]. Assuming the space has volume V, charge density ρ and closed surface area S, Lorentz force applied to the charges in a small volume dV by electric field intensity $\vec{E}$ and magnetic flux density $\vec{B}$ is given by [13]

$$\vec{f}dV = \rho dV \left(\vec{E} + \vec{v} \times \vec{B}\right), \tag{1}$$

where $\vec{v}$ is velocity of the charges ρdV. The work W_L done by Lorentz force in time range from t_0 to t may be written as

$$W_L = \int_V dV \int_{t_0}^{t} dt' \vec{f} \cdot \vec{v} = \int_V dV \int_{t_0}^{t} dt' \vec{J} \cdot \vec{E}, \tag{2}$$

in which $\vec{J} \equiv \rho\vec{v}$ is electric current density. Based on Maxwell's equations, equation (2) becomes

$$\begin{aligned} W_L &= \int_V dV \int_{t_0}^{t} dt' \vec{J} \cdot \vec{E} \\ &= -\oint_S dS \int_{t_0}^{t} dt' \left(\vec{E} \times \vec{H}\right) \cdot \hat{n} \\ &\quad - \int_V dV \int_{t_0}^{t} dt' \left(\vec{E} \cdot \frac{\partial \vec{D}}{\partial t'} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t'}\right), \end{aligned} \tag{3}$$

where $\vec{H}$ is magnetic field intensity, $\vec{D}$ is electric flux density, $\vec{S}_{Poynting} \equiv \vec{E} \times \vec{H}$ is time-dependent Poynting vector (TDPV), and $\hat{n}$ is an outward unit vector normal to the closed surface S. Both $W_e(t) = -\int_V dV \int_{t_0}^{t} dt' \vec{E} \cdot \frac{\partial \vec{D}}{\partial t'}$ and $W_m(t) = -\int_V dV \int_{t_0}^{t} dt' \vec{H} \cdot \frac{\partial \vec{B}}{\partial t'}$ may be taken as parts of work done by Lorentz force, respectively. According to relationship between work and alteration of electromagnetic energies, electric energy density $u_e(t)$ and magnetic energy density $u_m(t)$ are usually defined as $\frac{\partial u_e(t)}{\partial t} = \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$ and $\frac{\partial u_m(t)}{\partial t} = \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$, respectively. In a non-dispersive and lossless medium, there are $\frac{\partial \vec{D}}{\partial t} \cdot \vec{E} = \frac{\partial \vec{E}}{\partial t} \cdot \vec{D}$ and $\frac{\partial \vec{B}}{\partial t} \cdot \vec{H} = \frac{\partial \vec{H}}{\partial t} \cdot \vec{B}$, time-averaged electromagnetic energy density $\langle u_{e,m} \rangle \equiv \langle u_e + u_m \rangle$ is derived as [6,7]:

$$\langle u_{e,m} \rangle = \frac{1}{4}\left(\varepsilon\varepsilon_0 |E|^2 + \mu\mu_0 |H|^2\right). \tag{4}$$

It is noted that the well-known Poynting's theorem is usually written in the form [7,13]

$$\begin{aligned} \int_V dV \vec{J} \cdot \vec{E} &= -\oint_S dS \left(\vec{E} \times \vec{H}\right) \cdot \hat{n} \\ &\quad - \int_V dV \left(\vec{E} \cdot \frac{\partial \vec{D}}{\partial t'} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t'}\right). \end{aligned} \tag{5}$$

Comparing equation (3) with equation (5), it is stressed that Poynting's theorem focus on time-rate relationship among work done by Lorentz force, power flowing through the closed surface and changes of the electric and magnetic energies. Since Lorentz force is not necessarily conservative, the value of W_L may relate to the process of doing work. To clearly and generally demonstrate energy conversion relationships of electromagnetic fields in a lossy medium, we shall carefully treat equation (3) instead of equation (5).

For simplicity, we shall firstly pay our main attention on the terms of $W_e(t) = -\int_V dV \int_{t_0}^{t} dt' \vec{E} \cdot \frac{\partial \vec{D}}{\partial t'}$ and $W_m(t) = -\int_V dV \int_{t_0}^{t} dt' \vec{H} \cdot \frac{\partial \vec{B}}{\partial t'}$ for a single-frequency harmonic homogeneous plane wave (HHPW), which does not loose generality since any types of electromagnetic wave may be linearly composited by HHPWs. Choosing a frequency of $f = \omega/2\pi$, the electric field intensity and electric flux density of a HHPW in the medium having relative permittivity $\tilde{\varepsilon}$ and permeability $\tilde{\mu}$ can be, respectively, expressed as [13]:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{-\vec{k}'' \cdot (\vec{r} - \vec{r}_0)} \cos\left[\omega t - \vec{k}' \cdot (\vec{r} - \vec{r}_0)\right], \tag{6}$$

$$\vec{D}(\vec{r}, t) = |\tilde{\varepsilon}|\varepsilon_0 \vec{E}_0 e^{-\vec{k}'' \cdot (\vec{r} - \vec{r}_0)} \cos\left[\omega t - \vec{k}' \cdot (\vec{r} - \vec{r}_0) - \alpha_\varepsilon\right], \tag{7}$$

where $\vec{E}_0$ is electric field intensity of the wave at instant $t = 0$ and position $\vec{r}_0$, and $\tilde{\vec{k}} = \vec{k}' - i\vec{k}''$ is complex valued wave vector. Substituting equations (6) and (7) into $W_e(t) = -\int_V dV \int_{t_0}^{t} dt' \vec{E} \cdot \frac{\partial \vec{D}}{\partial t'}$, we have

$$\begin{aligned} W_e(t) &= -\int_V dV \frac{1}{4} |\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}'' \cdot (\vec{r} - \vec{r}_0)} \\ &\quad \times \left\{\cos\left[2\omega t - 2\vec{k}' \cdot (\vec{r} - \vec{r}_0) - \alpha_\varepsilon\right] \right. \\ &\quad \left. - \cos\left[2\omega t_0 - 2\vec{k}' \cdot (\vec{r} - \vec{r}_0) - \alpha_\varepsilon\right]\right\} \\ &\quad - \int_V dV \frac{1}{2} \omega |\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}'' \cdot (\vec{r} - \vec{r}_0)} \sin\alpha_\varepsilon (t - t_0). \end{aligned} \tag{8}$$

Analogously, we can obtain

$$W_m(t) = -\int_V dV \int_{t_0}^{t} dt' \left(\vec{H}\cdot\frac{\partial \vec{B}}{\partial t'}\right)$$
$$= -\int_V dV \frac{1}{4}|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}$$
$$\times\left\{\cos\left[2\omega t - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu\right]\right.$$
$$\left.-\cos\left[2\omega t_0 - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu\right]\right\}$$
$$-\int_V dV \frac{1}{2}\omega|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_\mu(t-t_0). \tag{9}$$

Apparently, both W_e and W_m include a time-dependent periodic term and a time-dependent linear one, respectively. Physically, periodic variation of the work indicates that energies are stored and then released by turns, i.e., the periodic terms relate to stored energies. Thus electric and magnetic stored energy densities may be, respectively, defined by

$$u_e(t) = \frac{1}{4}|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}$$
$$\times\left\{\cos\left[2\omega t - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\varepsilon\right]\right.$$
$$\left.-\cos\left[2\omega t_0 - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\varepsilon\right]\right\}$$
$$= \frac{1}{2}|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)},$$
$$\times\left\{\cos^2\left[\omega t - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\varepsilon/2\right]\right.$$
$$\left.-\cos^2\left[\omega t_0 - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\varepsilon/2\right]\right\} \tag{10}$$

$$u_m(t) = \frac{1}{4}|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}$$
$$\times\left\{\cos\left[2\omega t - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu\right]\right.$$
$$\left.-\cos\left[2\omega t_0 - 2\vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu\right]\right\}$$
$$= \frac{1}{2}|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}$$
$$\times\left\{\cos^2\left[\omega t - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu/2\right]\right.$$
$$\left.-\cos^2\left[\omega t_0 - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu/2\right]\right\}. \tag{11}$$

Let $\cos^2[\omega t_0 - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\varepsilon/2]=0$ and $\cos^2[\omega t_0 - \vec{k'}\cdot(\vec{r}-\vec{r}_0)-\alpha_\mu/2]=0$, time-averaged electric and magnetic stored energy densities are, respectively, obtained as:

$$\langle u_e\rangle = \frac{1}{4}|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}, \tag{12}$$
$$\langle u_m\rangle = \frac{1}{4}|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}. \tag{13}$$

Apparently, both $\langle u_e\rangle$ and $\langle u_m\rangle$ are always positive. In addition, it is noted from equations (10)–(13) that the minium value of the stored energy $u_{e(m)}(t)$ does not necessarily correspond to the case of both $E=0$ and $D=0$ ($H=0$ and $B=0$), which relates directly to non-synchronous variation of E and D (H and B) of electromagnetic wave in the lossy medium [10,11]. On the other hand, the time-dependent linear terms relate to lossy energies. The electric and magnetic lossy energy densities may be, respectively, defined as:

$$u_{e,less}(t) = \frac{1}{2}\omega|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_\varepsilon(t-t_0), \tag{14}$$
$$u_{m,less}(t) = \frac{1}{2}\omega|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_\mu(t-t_0). \tag{15}$$

Since the lossy energies in a range of time may be easily calculated by using time-rate of change of electric and magnetic lossy energy densities, time-rate of change of electric and magnetic lossy energy densities instead of time-averaged electric and magnetic lossy energy densities are, respectively, given by:

$$\frac{\partial u_{e,loss}(t)}{\partial t} = \frac{1}{2}\omega|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_\varepsilon, \tag{16}$$
$$\frac{\partial u_{m,loss}(t)}{\partial t} = \frac{1}{2}\omega|\tilde{\mu}|\mu_0 H_0^2 e^{-2\vec{k''}\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_\mu. \tag{17}$$

It is pointed out that definitions of $u_e(t)$,$u_m(t)$, $u_{e,loss}(t)$ and $u_{m,loss}(t)$, which are applicable for lossy media, are usually derived by adopting the equations of motion of electric and magnetic polarization [6,7]. The $u_{e(m),loss}(t)$ is directly taken as the term related to the electric (magnetic) damping one of the motion equation of electric (magnetic) polarization. In fact, definitions of lossy energy densities given in references [6,7], which are formed as functions of time integrals, may be further divided into a time-dependent periodic term and a time-dependent linear one after performing integral, respectively. Noting relationship between $\tilde{\varepsilon}(\tilde{\mu})$ and damping parameter $\Gamma_e(\Gamma_m)$, it is verified that time-dependent linear and periodic terms obtained by the two ways are identical to each other, respectively.

Adopting relation of $\tilde{E}(\vec{r},t)=\tilde{\eta}\tilde{H}(\vec{r},t)$ (where $\tilde{\eta}=\sqrt{\frac{\tilde{\mu}\mu_0}{\tilde{\varepsilon}\varepsilon_0}}=\sqrt{\frac{|\tilde{\mu}|\mu_0}{|\tilde{\varepsilon}|\varepsilon_0}}e^{-i\frac{\alpha_\mu-\alpha_\varepsilon}{2}}$ is impedance), equation (9) may be rewritten as:

$$W_m(t) = -\int_V dV \frac{1}{4}|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}''\cdot(\vec{r}-\vec{r}_0)} \times \left\{\cos\left[2\omega t - 2\vec{k}'\cdot(\vec{r}-\vec{r}_0) - \alpha_\varepsilon\right] - \cos\left[2\omega t_0 - 2\vec{k}'\cdot(\vec{r}-\vec{r}_0) - \alpha_\varepsilon\right]\right\} - \int_V dV \frac{1}{2}\omega|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}''\cdot(\vec{r}-\vec{r}_0)} \sin\alpha_\mu(t-t_0). \quad (18)$$

Equations (8) and (18) indicate that the periodic part of W_e always alters synchronously with that of W_m, although variation of electric and magnetic parameters of electromagnetic wave in the lossy medium is usually nonsynchronous. Thus time-averaged electromagnetic stored energy density $\langle u_{e,m}\rangle$ and time-rate of change of electromagnetic lossy energy density $\partial u_{e,m,loss}(t)/\partial t$ may be, respectively, proposed as

$$\langle u_{e,m}\rangle = \frac{1}{2}|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}''\cdot(\vec{r}-\vec{r}_0)}, \quad (19)$$

$$\frac{\partial u_{e,m,loss}(t)}{\partial t} = \frac{1}{2}\omega|\tilde{\varepsilon}|\varepsilon_0 E_0^2 e^{-2\vec{k}''\cdot(\vec{r}-\vec{r}_0)}(\sin\alpha_\mu + \sin\alpha_\varepsilon). \quad (20)$$

When both α_ε and α_μ equal zero, equation (19) reduces to equation (4).

It is known that time-averaged Poynting vector (TAPV) may be given by [13]:

$$\langle\vec{S}_{Ponting}\rangle = \frac{1}{2|\tilde{\eta}|}E_0^2 e^{-2\vec{k}''\cdot(\vec{r}-\vec{r}_0)} \cos\left(\frac{\alpha_\mu - \alpha_\varepsilon}{2}\right)\hat{e}_S, \quad (21)$$

where $\hat{e}_S$ is a unit vector with its direction parallels to that of $\langle\vec{S}_{Poynting}\rangle$. Under condition of $\int_V dV \int_{t_0}^t dt'\vec{J}\cdot\vec{E} = 0$, it is derived from equations (20) and (21), that

$$\oint_S dS\langle\vec{S}_{Poynting}\rangle\cdot\hat{n} = \int_V dV \frac{\partial u_{e,m,loss}(t)}{\partial t}. \quad (22)$$

Equation (22) further confirms the rationality of the definitions of stored and lossy energy densities obtained in this work.

In order to account for the dispersion, we consider the signal spectrum consisted of two discrete different frequencies $f_1 = \omega_1/2\pi$ and $f_2 = \omega_2/2\pi$ [14],

$$\vec{E}(\vec{r},t) = \vec{E}_{01}e^{-\vec{k}_1''\cdot(\vec{r}-\vec{r}_0)}\cos\left[\omega_1 t - \vec{k}_1'\cdot(\vec{r}-\vec{r}_0)\right] + \vec{E}_{02}e^{-\vec{k}_2''\cdot(\vec{r}-\vec{r}_0)}\cos\left[\omega_2 t - \vec{k}_2'\cdot(\vec{r}-\vec{r}_0)\right]. \quad (23)$$

It has been verified that the cross terms of the Poynting vector average to zero yielding [14]

$$\langle\vec{S}_{Ponting,total}\rangle = \langle\vec{S}_{Ponting,1}\rangle + \langle\vec{S}_{Ponting,2}\rangle. \quad (24)$$

In addition, $W_e(t) = -\int_V dV\int_{t_0}^t dt'\vec{E}\cdot\frac{\partial\vec{D}}{\partial t'}$ is given as:

$$\begin{aligned}
W_e(t) = &-\int_V dV\frac{1}{4}|\tilde{\varepsilon_1}|\varepsilon_0 E_{01}^2 e^{-2\vec{k}_1''\cdot(\vec{r}-\vec{r}_0)}\\
&\times\left\{\cos\left[2\omega_1 t - 2\vec{k}_1'\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right] - \cos\left[2\omega_1 t_0 - 2\vec{k}_1'\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right]\right\}\\
&-\int_V dV\frac{1}{2}\omega_1|\tilde{\varepsilon_1}|\varepsilon_0 E_{01}^2 e^{-2\vec{k}_1''\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_{\varepsilon 1}(t-t_0)\\
&-\int_V dV\frac{1}{4}|\tilde{\varepsilon_2}|\varepsilon_0 E_{02}^2 e^{-2\vec{k}_2''\cdot(\vec{r}-\vec{r}_0)}\\
&\times\left\{\cos\left[2\omega_2 t - 2\vec{k}_2'\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right] - \cos\left[2\omega_2 t_0 - 2\vec{k}_2'\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right]\right\}\\
&-\int_V dV\frac{1}{2}\omega_2|\tilde{\varepsilon_2}|\varepsilon_0 E_{02}^2 e^{-2\vec{k}_2''\cdot(\vec{r}-\vec{r}_0)}\sin\alpha_{\varepsilon 2}(t-t_0)\\
&-\int_V dV\frac{1}{2}|\tilde{\varepsilon_2}|\varepsilon_0 E_{01}E_{02} e^{-(\vec{k}_1''+\vec{k}_2'')\cdot(\vec{r}-\vec{r}_0)}\frac{\omega_2}{\omega_1+\omega_2}\\
&\times\left\{\cos\left[(\omega_1+\omega_2)t - (\vec{k}_1'+\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right] - \cos\left[(\omega_1+\omega_2)t_0 - (\vec{k}_1'+\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right]\right\}\\
&-\int_V dV\frac{1}{2}|\tilde{\varepsilon_2}|\varepsilon_0 E_{01}E_{02} e^{-(\vec{k}_1''+\vec{k}_2'')\cdot(\vec{r}-\vec{r}_0)}\frac{\omega_2}{\omega_2-\omega_1}\\
&\times\left\{\cos\left[(\omega_2-\omega_1)t - (\vec{k}_2'-\vec{k}_1')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right] - \cos\left[(\omega_2-\omega_1)t_0 - (\vec{k}_2'-\vec{k}_1')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 2}\right]\right\}\\
&-\int_V dV\frac{1}{2}|\tilde{\varepsilon_1}|\varepsilon_0 E_{01}E_{02} e^{-(\vec{k}_1''+\vec{k}_2'')\cdot(\vec{r}-\vec{r}_0)}\frac{\omega_1}{\omega_1+\omega_2}\\
&\times\left\{\cos\left[(\omega_1+\omega_2)t - (\vec{k}_1'+\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right] - \cos\left[(\omega_1+\omega_2)t_0 - (\vec{k}_1'+\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right]\right\}\\
&-\int_V dV\frac{1}{2}|\tilde{\varepsilon_1}|\varepsilon_0 E_{01}E_{02} e^{-(\vec{k}_1''+\vec{k}_2'')\cdot(\vec{r}-\vec{r}_0)}\frac{\omega_1}{\omega_1-\omega_2}\\
&\times\left\{\cos\left[(\omega_1-\omega_2)t - (\vec{k}_1'-\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right] - \cos\left[(\omega_1-\omega_2)t_0 - (\vec{k}_1'-\vec{k}_2')\cdot(\vec{r}-\vec{r}_0)-\alpha_{\varepsilon 1}\right]\right\}.
\end{aligned} \quad (25)$$

Apparently, the former four terms of equation (25) correspond to stored and lossy energy densities of $u_{e,1}(t), u_{e,lossy,1}(t), u_{e,2}(t)$ and $u_{e,lossy,2}(t)$ for the two independent electromagnetic waves, respectively, all of the later four cross terms of equation (25) are time-dependent periodic functions and not involved in the lossy energies. Analogously, we can verify that $W_m(t)$ has similar properties. Combining equations (3), (8), (22) and (24), it is concluded that the cross terms presented in equation (25) may be neglected when we address issues associated with TAPV and the corresponding stored and lossy energies.

3 On energy flow velocity

We shall pay some attention on energy flow velocity, which is usually defined as $\vec{v}_E = \frac{\langle \vec{S}_{Poynting} \rangle}{\langle u_{e,m} \rangle}$ [6]. Noting equations (19) and (21), we have

$$v_E = \frac{\cos\left(\frac{\alpha_\mu - \alpha_\varepsilon}{2}\right)}{\sqrt{|\tilde{\mu}\tilde{\varepsilon}|\mu_0\varepsilon_0}}. \tag{26}$$

Equation (26) offers a simple form of energy flow velocity and differs to the energy flow velocity formula of $v_E = (-1)^s \frac{2c\text{Re}(\sqrt{\tilde{\varepsilon}/\tilde{\mu}})}{(\varepsilon' + 2\omega\varepsilon''/\Gamma_e) + (\mu' + 2\omega\mu''/\Gamma_m)|\tilde{\varepsilon}/\tilde{\mu}|}$ marked as equation (22) in reference [6], where index s is equal to $+1$ for a right-handed medium and to -1 for a left-handed one, and c is light velocity in free space. The significant difference between two energy flow velocity formulas indicate that proper definitions of stored electromagnetic energy densities are important to study issues associated with energy flow velocity.

We shall focus on a special case: $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2}) = 0$. For passive medium, $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2}) = 0$ means that $\alpha_{\varepsilon(\mu)} = 0$ and $\alpha_{\mu(\varepsilon)} = \pi$, thus $\cos(\frac{\alpha_\mu + \alpha_\varepsilon}{2}) = 0$, i.e., $\text{Re}(\tilde{k}) = 0$ but $\text{Im}(\tilde{k}) \neq 0$, propagation of electromagnetic wave in this medium is forbidden [13]. However, for active medium, conditions of $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2}) = 0$ and $\sin(\frac{\alpha_\mu + \alpha_\varepsilon}{2}) = 0$ (i.e., $\text{Re}(\tilde{k}) \neq 0$ but $\text{Im}(\tilde{k}) = 0$) may be satisfied as $\alpha_{\varepsilon(\mu)} = \frac{\pi}{2}$ and $\alpha_{\mu(\varepsilon)} = \frac{3\pi}{2}$. It is found from equations (12), (13) and (19) that, here, $\langle u_e \rangle$, $\langle u_m \rangle$ and $\langle u_{e,m} \rangle$ do not necessarily equal zero. In addition, $\partial u_{e,m,loss}(t)/\partial t = 0$ due to that of $\alpha_{\varepsilon(\mu)} = \frac{\pi}{2}$ and $\alpha_{\mu(\varepsilon)} = \frac{3\pi}{2}$, which means balance between electric loss (gain) and magnetic gain (loss). On the other hand, in this case, direction of TDPV alter with change of time, i.e., time-dependent energy flow shift forward and then turn backward periodically, which leads to that $\langle \vec{S}_{Ponting} \rangle = 0$ and light local in a space range of $\frac{\lambda}{4}$ (λ is wavelength of light in this medium) along the propagation direction, i.e., light seem to stop steadily.

Furthermore, we shall simply investigate cases associated with changes of the sign of $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2})$. According to equation (21), propagation direction of TAPV may alter with change of the sign of $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2})$. Thus propagation direction of TAPV in an active medium may be controlled by adjusting either permittivity or permeability.

Finally, we shall suggest a possible experiment to verify our theoretical predictions by adopting microwave instead of light. Recently, active microwave negative-index metamaterial transmission line with gain has been realized [9]. The unit cell of adopted general metamaterial composite right-handed or left-handed transmission line (CRLH TL) includes a series inductance (L_R) and resistance (R), a shunt capacitance (C_R) and conductance (G), a series capacitance (C_L) and a shunt inductance (L_L), the tunnel diode device connected in series is used to implement the active CRLH TL. The effective permittivity and permeability of the CRLH TL may be given as [8,9]:

$$\tilde{\varepsilon}_{\text{eff}} = \left[\left(C_R - \frac{1}{\omega^2 L_L}\right) + i\frac{G}{\omega}\right]/p, \tag{27}$$

$$\tilde{\mu}_{\text{eff}} = \left[\left(L_R - \frac{1}{\omega^2 C_L}\right) + i\frac{R}{\omega}\right]/p, \tag{28}$$

where R is negative because of incorporation of the tunnel diode and p is the equivalent section length approximated by the unit cell. Apparently, when appropriate values of L_R, C_R, C_L, L_L and ω are chosen, both $C_R - \frac{1}{\omega^2 L_L} = 0$ and $L_R - \frac{1}{\omega^2 C_L} = 0$ may be obtained simultaneously, i.e., conditions of $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2}) = 0$ and $\sin(\frac{\alpha_\mu + \alpha_\varepsilon}{2}) = 0$ are satisfied, microwave may stop steadily in this transmission line. In addition, it is clear that sign of $\cos(\frac{\alpha_\mu - \alpha_\varepsilon}{2})$, and then direction of TAPV, may be altered by adjusting values of L_R, C_R, C_L or L_L, respectively.

4 Conclusions

In summary, on the basis of work done by Lorentz force and Maxwell's equations, new definitions of electromagnetic stored and lossy energy densities are proposed, respectively. Then properties of energy flow velocity in lossy media are investigated. Interestingly, it is predicted that, in an active medium, light may stop steadily and TAPV direction of light is controllable. To the best of our knowledge, this is an alternative procedure proposed to stop light. These results may open the way to a multitude of hybrid, optoelectronic devices to be used in "quantum information" processing, communication network and signal processors. Confirmations about the expected properties should motivate further theoretical progress.

Author contribution statement

All authors contributed equally to the paper.

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